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J.2475 If $t > 0, u \geq 0$, then in ΔABC holds:

$$(ab)^{t(u+1)} + (bc)^{t(u+1)} + (ca)^{t(u+1)} \geq 4^{t(u+1)} 3^{1-t(u+1)} \left(\sqrt{3}\right)^{t(u+1)} F^{t(u+1)}$$

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Solution by Titu Zvonaru-Romania

Applying $AM - GM$ and Carlitz's inequality $(abc)^{2/3} \geq \frac{4}{3}\sqrt{3}F$,

it follows that

$$(ab)^{t(u+1)} + (bc)^{t(u+1)} + (ca)^{t(u+1)} \geq 3(abc)^{\frac{2t(u+1)}{3}} =$$

$$= 3\left((abc)^{\frac{2}{3}}\right)^{t(u+1)} \geq 3\left(\frac{4}{3}\sqrt{3}F\right)^{t(u+1)} = 4^{t(u+1)} 3^{1-t(u+1)} \left(\sqrt{3}\right)^{t(u+1)} F^{t(u+1)}.$$

Equality holds if and only if ΔABC is equilateral.