ROMANIAN MATHEMATICAL MAGAZINE

J.2476 If $x, y, z > 0$, then in ΔABC holds:

$$\frac{ae^{x^2}}{(y+z)h_a}+\frac{be^{y^2}}{(z+x)h_b}+\frac{ce^{z^2}}{(x+y)h_c}>2\sqrt{3}$$

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Solution by Titu Zvonaru-Romania

For $t > 0$ we consider the function $f(t) = e^t - 1 - t$.

We have $f'(t) = e^t - 1, f''(t) = e^t > 0$. Yields that f' is an increasing function, hence $f'(t) > f'(0) = 0$. We deduce that the function f is increasing; then $f(t) > f(0) = 0$.

For $t = x^2$, by $AM - GM$ it results that $e^{x^2} > 1 + x^2 \geq 2x \quad (1)$

We have $ah_a = bh_b = ch_c = 2F$. Applying inequality (1) and the Tsintsifas' inequality $\frac{x}{y+z}a^2 + \frac{y}{z+x}b^2 + \frac{z}{x+y}c^2 \geq 2\sqrt{3F}$, it follows that

$$\frac{ae^{x^2}}{(y+z)h_a}+\frac{be^{y^2}}{(z+x)h_b}+\frac{ce^{z^2}}{(x+y)h_c}=\frac{a^2e^{x^2}}{(y+z)ah_a}+\frac{b^2e^{y^2}}{(z+x)bh_b}+\frac{c^2e^{z^2}}{(x+y)ch_c}>$$

$$>\frac{1}{2F}\left(\frac{2xa^2}{y+z}+\frac{2yb^2}{z+x}+\frac{2zc^2}{x+y}\right)=\frac{1}{F}\left(\frac{x}{y+z}a^2+\frac{y}{z+x}b^2+\frac{z}{x+y}c^2\right)\geq\frac{1}{F}\left(2\sqrt{3}F\right)=2\sqrt{3}$$