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J.2477 If $x, y, z > 0$, then in ΔABC holds:

$$\frac{a^2 e^{x^2}}{y+z} + \frac{b^2 e^{y^2}}{z+x} + \frac{c^2 e^{z^2}}{x+y} > 4\sqrt{3}F$$

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Solution by Titu Zvonaru-Romania

For $t > 0$ we consider the function $f(t) = e^t - 1 - t$.

We have $f'(t) = e^t - 1, f''(t) = e^t > 0$. Yields that f' is a increasing function, hence $f'(t) > f'(0) = 0$.

We deduce that the function f is increasing; then $f(t) > f(0) = 0$.

For $t = x^2$, by $AM - GM$ it results that $e^{x^2} > 1 + x^2 \geq 2x \quad (1)$

Applying inequality (1) and the Tsintsifas inequality:

$\frac{x}{y+z}a^2 + \frac{y}{z+x}b^2 + \frac{z}{x+y}c^2 \geq 2\sqrt{3}F$, it follows that:

$$\frac{a^2 e^{x^2}}{y+z} + \frac{b^2 e^{y^2}}{z+x} + \frac{c^2 e^{z^2}}{x+y} > 2\left(\frac{x}{y+z}a^2 + \frac{y}{z+x}b^2 + \frac{z}{x+y}c^2\right) \geq 2(2\sqrt{3}F) = 4\sqrt{3}F$$