ROMANIAN MATHEMATICAL MAGAZINE

J.2496 Prove that the equation $y^2 = 25x^2 + 26$ does not have solutions integer numbers.

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Solution by Titu Zvonaru-Romania

The equation is equivalent to $(y - 5x)(y + 5x) = 26$.

Since $y - 5x + y + 5x = 2y$, it results that the integer numbers

$y - 5x$ and $y + 5x$ have the same parity.

Because $26 = 1 \cdot 26 = 2 \cdot 13$, we do not obtain integer solutions.