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J.2505 Let by $n \in N, n \geq 2$. Prove that:

$$\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}>\frac{n-1}{n+2}$$

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We prove by induction.

For $n=2$ we have to prove that $\frac{1}{3}>\frac{1}{4}$, which is true.

Suppose that

$$\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}>\frac{n-1}{n+2} \qquad (1)$$

We have to prove that

$$\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}+\frac{1}{2n+1}>\frac{n}{n+3} \qquad (2)$$

Using (1) we obtain

$$\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}+\frac{1}{2n+1}>\frac{n-1}{n+2}+\frac{1}{2n+1}.$$

It remains to prove

$$\frac{n-1}{n+2}+\frac{1}{2n+1}>\frac{n}{n+3}$$

$$(n-1)(n+3)(2n+1)+(n+1)(n+3)>n(n+2)(2n+1)$$

$$2n^3+4n^2-6n+n^2+2n-3+n^2+4n+3>2n^3+4n^2+n^2+2n$$

$$n^2>2n,$$

obviously true. It follows that (2) is true; by induction it results that (1) is true.