ROMANIAN MATHEMATICAL MAGAZINE

J.2507 Prove that $E(x) = x^{12} - x^9 + x^4 - x + 1 > 0$ for any $x \in R$.

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If $x \leq 0$ then $-x^9 \geq 0, -x \geq 0$, hence $E(x) > 0$.

Writing $E(x) = x^{12} + x^4(1 - x^5) + 1 - x > 0$,

we deduce that if $x \in (0, 1]$ then $E(x) > 0$.

For $x > 1$, we write $E(x) = x^9(x^3 - 1) + x(x^3 - 1) + 1$; it follows that $E(x) > 0$.