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J.2517 In triangle ABC the bisectors AD and CE intersect at point I. The points B, D, I, E lies themselves on a circle. Find the measure of angle B.

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Solution by Titu Zvonaru-Romania

Using bisector's theorem we have

$$AD = \frac{2bc\cos\frac{A}{2}}{b+c}, AI = \frac{bc\cos\frac{A}{2}}{s}, AE = \frac{bc}{a+b}.$$

Applying the power of point A with respect to the circumcircle of quadrilateral $BDIE$ it follows that

$$AI \cdot AD = AE \cdot AB$$

which is equivalent to

$$\frac{bc\cos\frac{A}{2}}{s} \cdot \frac{2bc\cos\frac{A}{2}}{b+c} = \frac{bc}{a+b} \cdot c \Leftrightarrow \frac{2bs(s-a)}{sbc(b+c)} = \frac{1}{a+b} \Leftrightarrow$$

$$\Leftrightarrow 2(a+b)(s-a) = c(b+c) \Leftrightarrow (a+b)(b+c-a) = c(b+c)$$

$$\Leftrightarrow ab + ac - a^2 + b^2 + bc - ab = bc + c^2 \Leftrightarrow ac = a^2 + c^2 - b^2$$

$$\Leftrightarrow ac = 2ac\cos B \Leftrightarrow \cos B = \frac{1}{2},$$

hence $B = 60°$.