

ROMANIAN MATHEMATICAL MAGAZINE

J.2519 Find $m, n \in R$ such that $E(x) = x^4 + mx^3 + nx^2 - 8x + 1$ be perfect square.

Proposed by Gilena Dobrică – Romania

Solution by Titu Zvonaru-Romania

We have to determine $p, q \in R$ such that

$$x^4 + mx^3 + nx^2 - 8x + 1 = (x^2 + px + q)^2$$

$$x^4 + mx^3 + nx^2 - 8x + 1 = x^4 + 2px^3 + (p^2 + 2q)x^2 + 2pqx + q^2$$

It follows that $m = 2p, n = p^2 + 2q, -8 = 2pq, 1 = q^2$. We have two possibilities:

$$q = 1, p = -4, n = 18, m = -8$$

$$q = -1, p = 4, n = 14, m = 8$$

Since

$$\begin{aligned}(x^2 - 4x + 1)^2 &= x^4 - 8x^3 + 18x^2 - 8x + 1, (x^2 + 4x - 1)^2 \\ &= x^4 + 8x^3 + 14x^2 - 8x + 1,\end{aligned}$$

yields that $(m, n) = (-8, 18), (8, 14)$.