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J.2538 If $1 \leq x \leq 2 \leq y \leq 3 \leq z$ then

$$\frac{2}{z} + \frac{2z}{3} > \frac{x}{2} + \frac{1}{x} + \frac{y}{6} + \frac{1}{y}$$

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Since $1 \leq x \leq 2$ we have

$$(x-1)(x-2) \leq 0 \Leftrightarrow x^2 + 2 \leq 3x \Leftrightarrow \frac{x}{2} + \frac{1}{x} \leq \frac{3}{2} \quad (1)$$

Since $2 \leq y \leq 3$ we obtain

$$(y-2)(y-3) \leq 0 \Leftrightarrow y^2 + 6 \leq 5y \Leftrightarrow \frac{y}{6} + \frac{1}{y} \leq \frac{5}{6} \quad (2)$$

By (1) and (2) yields that it suffices to prove that

$$\frac{2}{z} + \frac{2z}{3} \geq \frac{3}{2} + \frac{5}{6} \Leftrightarrow \frac{2z^2+6}{3z} \geq \frac{14}{6} \Leftrightarrow 2z^2 + 6 \geq 7z \Leftrightarrow (z-2)(2z-3) \geq 0,$$

which is true because $z \geq 3$. The inequality is strict.