ROMANIAN MATHEMATICAL MAGAZINE

J.2554 Solve for real numbers:

$$5x+13=2y+3z$$

$$2x^2+y^2+z^2=27$$

$$x(y+z)=7$$

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By $2y+3z=5x+13$ and $y+z=\frac{7}{x}$ we obtain

$$y=\frac{21}{x}-5x-13, z=5x+13-\frac{14}{x} \quad (1)$$

Using (1), the second equation is

$$2x^2+\left(\frac{21}{x}-5x-13\right)^2+\left(5x+13-\frac{14}{x}\right)^2=27$$

$$52x^4+260x^3-39x^2-910x+637=0$$

$$4x^4+20x^3-3x^2-70x+49=0 \quad (2)$$

Applying Horner, it results

	4	20	-3	-70	49
1	4	24	21	-49	0
1	4	28	49	0	

It follows that the equation (2) becomes

$$(x-1)^2(4x^2+28x+49)=0 \Leftrightarrow (x-1)^2(2x+7)^2=0.$$

The solutions are $(x,y,z)=(1,3,4),\left(-\frac{7}{2},-\frac{3}{2},-\frac{1}{2}\right)$.