ROMANIAN MATHEMATICAL MAGAZINE

J.2555 If $x, y, z \geq 0$, then:

$$x^3 + y^3 + 2z^3 \geq z(xz + yz + 2xy)$$

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Applying $AM - GM$ inequality it follows that

$$\frac{x^3}{3} + \frac{z^3}{3} + \frac{z^3}{3} \geq xz^2, \frac{y^3}{3} + \frac{z^3}{3} + \frac{z^3}{3} \geq yz^2, 2\left(\frac{x^3}{3} + \frac{y^3}{3} + \frac{z^3}{3}\right) \geq 2xyz.$$

Adding these three inequalities, we obtain

$$x^3 + y^3 + 2z^3 \geq z(xz + yz + 2xy).$$

Equality holds if and only if $x = y = z$.