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J.2581 In triangle ABC holds:

$$\sqrt{2}a\cos\frac{B}{2}\cos\frac{C}{2}=s\ \Rightarrow \sec(2B)+\tan(2\mathrm{B})=\frac{c+b}{c-b}$$

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Since $\cos\frac{B}{2}=\sqrt{\frac{s(s-b)}{ac}}, \cos 2x=\frac{1-\tan^2 x}{1+\tan^2 x}, \tan 2x=\frac{2\tan x}{1-\tan^2 x}$ we obtain:

$$\sqrt{2}a\cos\frac{B}{2}\cos\frac{C}{2}=s\ \Rightarrow 2a^2\cdot\frac{s(s-b)}{ac}\cdot\frac{s(s-c)}{ab}=s^2\Rightarrow 2(s-b)s-c)=bc$$

$$\Rightarrow (a-b+c)(a+b-c)=2bc\Rightarrow a^2-b^2+2bc-c^2=2bc\Rightarrow$$

$$a^2=b^2+c^2\Rightarrow A=90^\circ\Rightarrow \tan B=\frac{b}{c}\Rightarrow$$

$$\sec(2B)+\tan(2B)=\frac{1}{\cos(2B)}+\tan(2\mathrm{B})=$$

$$=\frac{1+\tan^2 B}{1-\tan^2 B}+\frac{2\tan B}{1-\tan^2 B}=\frac{(1+\tan B)^2}{1-\tan^2 B}=\frac{1+\tan B}{1-\tan B}=\frac{1+\frac{b}{c}}{1-\frac{b}{c}}=\frac{c+b}{c-b}$$