J.2594 Solve for real numbers

$$\frac{1}{1 + \tan^4 x} + \frac{1}{10} = \frac{2}{1 + 3 \tan^2 x}$$

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We denote $t = \tan^2 x$. We have the equation:

$$\frac{1}{1 + t^2} + \frac{1}{10} = \frac{2}{1 + 3t} \Leftrightarrow 10 + 30t + 1 + t^2 + 3t + 3t^3 = 20 + 20t^2 \Leftrightarrow$$

$$3t^3 - 19t^2 + 33t - 9 = 0 \Leftrightarrow (3t - 1)(t - 3)^2 = 0.$$

It follows that $\tan x = \pm \frac{1}{\sqrt{3}}, \tan x = \pm\sqrt{3}$; the solutions are

$$x = \pm 30° + 180°k, x = \pm 60° + 180°k,$$

where k is an integer numbers.