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J.2629 If $a,b,c,>0$ then:

$$\frac{a}{(1936b+86c)^m}+\frac{b}{(1936c+86a)^m}+\frac{c}{(1936a+86b)^m}\ge\frac{\left(\sqrt{3}\right)^{m+1}}{2022^m}(ab+bc+ca)^{\frac{1-m}{2}}$$

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Applying Radon's inequality and the known inequality $(a+b+c)^2\ge3(ab+bc+ca)$, it follows that:

$$\frac{a}{(1936b+86c)^m}+\frac{b}{(1936c+86a)^m}+\frac{c}{(1936a+86b)^m}\ge$$

$$\ge\frac{a^{m+1}}{(1936ab+86ca)^m}+\frac{b^{m+1}}{(1936bc+86ab)^m}+\frac{c^{m+1}}{(1936ca+86bc)^m}$$

$$\ge\frac{(a+b+c)^{m+1}}{\big(1936(ab+bc+ca)+86(ab+bc+ca)\big)^m}=\frac{(a+b+c)^{m+1}}{2022^m(ab+bc+ca)^m}\ge$$

$$\ge\frac{\left(\sqrt{3}\right)^{m+1}(ab+bc+ca)^{\frac{m+1}{2}}}{2022^m(ab+bc+ca)^m}=\frac{\left(\sqrt{3}\right)^{m+1}}{2022^m}(ab+bc+ca)^{\frac{1-m}{2}}.$$

Equality holds if and only if $a=b=c$.