ROMANIAN MATHEMATICAL MAGAZINE

S.2350 If $m \geq 0$ and $x, y, z > 0$, then in ΔABC holds:

$$3m + \left(\frac{xa^2}{y+z}\right)^{m+1} + \left(\frac{yb^2}{z+x}\right)^{m+1} + \left(\frac{zc^2}{x+y}\right)^{m+1} \geq 2(m+1)\sqrt{3} \cdot F$$

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By Power Mean inequality we have:

$$\left(\frac{x^{m+1} + y^{m+1} + z^{m+1}}{3}\right)^{\frac{1}{m+1}} \geq \frac{x+y+z}{3} \Leftrightarrow$$

$$\Leftrightarrow 3^m(x^{m+1} + y^{m+1} + z^{m+1}) \geq (x+y+z)^{m+1} \quad (1)$$

Applying (1), Tsintsifas inequality $\frac{x}{y+z}a^2 + \frac{y}{z+x}b^2 + \frac{z}{x+y}c^2 \geq 2\sqrt{3}F$ and

$AM-GM$ inequality, it follows that:

$$3m + \left(\frac{xa^2}{y+z}\right)^{m+1} + \left(\frac{yb^2}{z+x}\right)^{m+1} + \left(\frac{zc^2}{x+y}\right)^{m+1} \geq$$

$$\geq 3m + \frac{1}{3^m}\left(\frac{x}{y+z}a^2 + \frac{y}{z+x}b^2 + \frac{z}{x+y}c^2\right)^{m+1} \geq$$

$$\geq \underbrace{3+3+\cdots+3}_{m\ cifre} + \frac{1}{3^m}\left(2\sqrt{3}F\right)^{m+1} \geq (m+1)\left(3 \cdot 3 \cdot \ldots \cdot 3 \cdot \frac{1}{3^m}\left(2\sqrt{3}F\right)^{m+1}\right)^{\frac{1}{m+1}}$$

$$= 2(m+1)\sqrt{3} \cdot F.$$

Equality holds if and only if ΔABC is equilateral, $x = y = z$ and $3 = \frac{1}{3^m}\left(2\sqrt{3} \cdot F\right)^{m+1}$.