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S.2351 If $x, y, z > 0$ then in triangle ABC holds

$$\frac{xa}{\sqrt{yz}} + \frac{yb}{\sqrt{zx}} + \frac{zc}{\sqrt{xy}} \geq 2 \cdot 3^{\frac{3}{4}} \cdot \sqrt{F}$$

Proposed by D.M.Bătinețu-Giurgiu, Cristina Ene – Romania

Solution by Titu Zvonaru-Romania

Applying $AM - GM$ inequality and Carlitz's inequality:

$$(abc)^{2/3} \geq \frac{4}{\sqrt{3}} F, \text{ it follows that:}$$

$$\frac{xa}{\sqrt{yz}} + \frac{yb}{\sqrt{zx}} + \frac{zc}{\sqrt{xy}} \geq 3 \left(\frac{xa}{\sqrt{yz}} \cdot \frac{yb}{\sqrt{zx}} \cdot \frac{zc}{\sqrt{xy}} \right)^{\frac{1}{3}} = 3(abc)^{\frac{1}{3}} \geq 3 \left(\frac{4}{\sqrt{3}} F \right)^{\frac{1}{2}} = 2 \cdot 3^{\frac{3}{4}} \cdot \sqrt{F}.$$

Equality holds if and only if triangle ABC and $x = y = z$.