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S.2355 If $x, y, z > 0$ then in ΔABC holds:

$$x\left(\frac{1}{y}+\frac{1}{z}\right)\cdot\frac{a}{h_b}+y\left(\frac{1}{z}+\frac{1}{x}\right)\cdot\frac{b}{h_c}+z\left(\frac{1}{x}+\frac{1}{y}\right)\cdot\frac{c}{h_a}\geq 4\sqrt{3}$$

Proposed by D.M.Bătinețu-Giurgiu, Oana Simona Dascălu - Romania

Solution by Titu Zvonaru-Romania

Using $AM-GM$ inequality and Carlitz' inequality $(abc)^{2/3}\geq\frac{4}{3}\sqrt{3}F$ we obtain:

$$x\left(\frac{1}{y}+\frac{1}{z}\right)\cdot\frac{a}{h_b}+y\left(\frac{1}{z}+\frac{1}{x}\right)\cdot\frac{b}{h_c}+z\left(\frac{1}{x}+\frac{1}{y}\right)\cdot\frac{c}{h_a}\geq$$

$$\overset{AM-GM}{\geq}\frac{2x}{\sqrt{yz}}\cdot\frac{ab}{2F}+\frac{2y}{\sqrt{zx}}\cdot\frac{bc}{2F}+\frac{2z}{\sqrt{xy}}\cdot\frac{ca}{2F}\geq$$

$$\overset{AM-GM}{\geq}\frac{3}{2F}\left(\frac{2x}{\sqrt{yz}}\cdot\frac{2y}{\sqrt{zx}}\cdot\frac{2z}{\sqrt{xy}}\cdot(abc)^2\right)^{\frac{1}{3}}\overset{CARLITZ}{\geq}\frac{6}{2F}\cdot\frac{4}{3}\sqrt{3}F\geq 4\sqrt{3}$$

Equality holds if and only if ΔABC is equilateral.