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S.2360 If $m \geq 0, x, y, z > 0$ then in triangle ABC holds:

$$\frac{x \cdot a^{m+1}}{\sqrt{yz}} + \frac{y \cdot b^{m+1}}{\sqrt{zx}} + \frac{z \cdot c^{m+1}}{\sqrt{xy}} \geq 2^{m+1} \cdot 3^{\frac{3-m}{4}} \cdot \left(\sqrt{F}\right)^{m+1}$$

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Applying $AM - GM$ inequality and Carlitz's inequality $(abc)^{2/3} \geq \frac{4}{\sqrt{3}}F$, it follows that

$$\frac{xa^{m+1}}{\sqrt{yz}} + \frac{yb^{m+1}}{\sqrt{zx}} + \frac{zc^{m+1}}{\sqrt{xy}} \geq 3\left(\frac{xa^{m+1}}{\sqrt{yz}} \cdot \frac{yb^{m+1}}{\sqrt{zx}} \cdot \frac{zc^{m+1}}{\sqrt{xy}}\right)^{\frac{1}{3}} = 3(abc)^{\frac{m+1}{3}} \geq$$

$$\geq 3\left(\frac{4}{\sqrt{3}}F\right)^{\frac{m+1}{2}} = 2^{m+1} \cdot 3^{\frac{3-m}{4}} \cdot \left(\sqrt{F}\right)^{m+1}.$$

Equality holds if and only if triangle ABC and $x = y = z$.

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