ROMANIAN MATHEMATICAL MAGAZINE

S.2361 If $M \in Int(\Delta ABC), d_a = d(M, BC), d_b = d(M, CA), d_c = d(M, AB)$ **then:**

$$\frac{a^5}{d_a^3} + \frac{b^5}{d_b^3} + \frac{c^5}{d_c^3} \geq 108F$$

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We have $ad_a + bd_b + cd_c = 2F$

$$\left((ad_a)(bd_b)(cd_c)\right) \overset{AM-GM}{\leq} \frac{(ad_a + bd_b + cd_c)^3}{27} = \frac{8F^3}{27} \quad (1)$$

Using $AM - GM$ inequality and Carlitz' inequality $(abc)^{2/3} \geq \frac{4}{3}\sqrt{3}F$, it follows that:

$$\frac{a^5}{d_a^3} + \frac{b^5}{d_b^3} + \frac{c^5}{d_c^3} = \frac{a^8}{a^3 d_a^3} + \frac{b^8}{b^3 d_b^3} + \frac{c^8}{c^3 d_c^3} \overset{AM-GM}{\geq} 3\left(\frac{(abc)^8}{\left((ad_a)(bd_b)(cd_c)\right)^3}\right)^{1/3} \geq$$

$$\overset{(1),CARLITZ}{\geq} 3 \cdot \frac{256F^4}{9} \cdot \frac{27}{8F^3} = 288F.$$

Equality holds if and only if ΔABC is equilateral and M is the circumcenter.