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**S.2362 If $M \in Int(\Delta ABC), d_a = d(M,BC), d_b = d(M,CA), d_c = d(M,AB)$ then:**

$$\frac{a^2b}{d_b}+\frac{b^2c}{d_c}+\frac{c^2a}{d_a} \ge 24F$$

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*Solution by Titu Zvonaru-Romania*

We have $ad_a + bd_b + cd_c = 2F$.

Using Bergström inequality and Gordon's inequality, it follows that

$$\frac{a^2b}{d_b}+\frac{b^2c}{d_c}+\frac{c^2a}{d_a}=\frac{a^2b^2}{bd_b}+\frac{b^2c^2}{cd_c}+\frac{c^2a^2}{ad_a} \ge$$

$$\overset{BERGSTROM}{\ge} \frac{(ab+bc+ca)^2}{ad_a+bd_b+cd_c} \overset{GORDON}{\ge} \frac{\left(4\sqrt{3}F\right)^2}{ad_a+bd_b+cd_c} \ge \frac{48F}{2F} = 24F.$$

Equality holds if and only if $\Delta ABC$ is equilateral.