S.2367 Solve for real numbers:

$$x^3(x+1)^3 + 8x^3 + 8 = 12x^3(x+1)$$

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Solution by Titu Zvonaru-Romania

Using Horner, we obtain

$$x^3(x+1)^3 + 8x^3 + 8 - 12x^3(x+1) = x^6 + 3x^5 + 3x^4 - 3x^3 - 12x^2 + 8$$
$$= (x-1)^2(x+1)(x+2)(x^2+2x+4).$$

It results that real roots are $x_1 = x_2 = 1, x_3 = -1, x_4 = -2$.