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**S.2374** $AA', BB', CC'$ – internal bisectors of $\Delta ABC$ with $I$ incenter. Prove that:

$$216[IBA'][ICB'][IAC'] \le F^3$$

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By bisector theorem we have $IA = \frac{bc\cos\frac{A}{2}}{s}, AC' = \frac{bc}{a+c}$. Yields that

$$[IAC'] = \frac{b^2c^2\sin\frac{A}{2}\cos\frac{A}{2}}{2s(a+c)} = \frac{b^2c^2\sin A}{4s(a+c)} = \frac{bcF}{2s(a+c)}.$$

Using $AM-GM$ inequality we obtain $(a+b)(b+c)(c+a) \ge 8abc$ and

$(a+b+c)^3 \ge 27abc$ . It follows that:

$$216[IBA'][ICB'][IAC'] = \frac{216a^2b^2c^2F^3}{8s^3(a+b)(b+c)(c+a)} \le$$

$$\le \frac{216a^2b^2c^2F^3}{8abc(a+b+c)^3} \le \frac{27abcF^3}{27abc} = F^3$$

Equality holds if and only if $a = b = c$ that is if and only if $\Delta ABC$ is equilateral.