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S.2382 If in $\Delta ABC, a \neq b \neq c \neq a$, then

$$\sum_{cyc} \frac{b+c}{b+c-a} > \frac{6s}{w_a + w_b + w_c}$$

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Applying Bergstrom's inequality and the well-known inequality:

$ab + bc + ca \leq a^2 + b^2 + c^2$, **we obtain:**

$$\sum_{cyc} \frac{b+c}{b+c-a} - 3 = \sum_{cyc} \left(\frac{b+c}{b+c-a} - 1 \right) = \sum_{cyc} \frac{a}{b+c-a} =$$

$$= \sum_{cyc} \frac{a^2}{ab+ac-a^2} \geq \frac{(a+b+c)^2}{2(ab+bc+ca) - (a^2+b^2+c^2)} =$$

$$= \frac{3(a+b+c)^2}{6(ab+bc+ca) - 3(a^2+b^2+c^2)} \geq \frac{3(a+b+c)^2}{2(ab+bc+ca) + a^2+b^2+c^2} = 3,$$

hence

$$\sum_{cyc} \frac{b+c}{b+c-a} \geq 6.$$

It remains to prove that:

$$6 > \frac{6s}{w_a + w_b + w_c} \Leftrightarrow w_a + w_b + w_c > s \qquad (1)$$

The inequality (1) is the item 8.9 from $[1]$.

$[1]$ **O. Bottema, *Geometric Inequalities*, Groningen 1969**