ROMANIAN MATHEMATICAL MAGAZINE

S.2391 In $\triangle ABC$ the following relationship holds:

$$(2a+b)(2c+b)+(2b+c)(2a+c)+(2c+a)(2b+a) \le 81R^2$$

Proposed by Daniel Sitaru, Elena Nedelcu - Romania

Solution by Titu Zvonaru-Romania

The given inequality is equivalent to:

$$a^2 + b^2 + c^2 + 8(ab + bc + ca) \le 81R^2$$

which follows by $a^2+b^2+c^2\leq 9R^2$ (item 5. 13 from [1]) and $ab+bc+ca\leq 9R^2$ (item 5. 16 from [1]).

[1] O. Bottema, Geometric Inegalities, Groningen 1969