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S.2397 In ΔABC the following relationship holds:

$$\frac{a}{\sin\frac{A}{3}}+\frac{b}{\sin\frac{B}{3}}+\frac{c}{\sin\frac{C}{3}}<18R$$

Proposed by Daniel Sitaru, Laura Zaharia -Romania

Solution by Titu Zvonaru-Romania

Since by theorem of sines we have $a=2R\sin A$ and

$\sin A=\sin\frac{A}{3}\left(3-4\sin^2\frac{A}{3}\right)$, we have to prove $9-4\left(\sin^2\frac{A}{3}+\sin^2\frac{B}{3}+\sin^2\frac{C}{3}\right)<9$,

which is obviously true.