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**S.2406 In $\Delta ABC$ the following relationship holds**

$$\frac{a^2m_b}{\sqrt{m_cm_a}}+\frac{b^2m_c}{\sqrt{m_am_b}}+\frac{c^2m_a}{\sqrt{m_bm_c}}\geq 4\sqrt{3}F$$

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Using $AM-GM$ inequality and Carlitz's inequality $(abc)^{2/3}\geq\frac{4}{3}F\sqrt{3}$,

we obtain:

$$\frac{a^2m_b}{\sqrt{m_cm_a}}+\frac{b^2m_c}{\sqrt{m_am_b}}+\frac{c^2m_a}{\sqrt{m_bm_c}}\geq 3\left(\frac{m_b}{\sqrt{m_cm_a}}\cdot\frac{m_c}{\sqrt{m_am_b}}\cdot\frac{m_a}{\sqrt{m_bm_c}}a^2b^2c^2\right)^{\frac{1}{3}}\geq$$

$$\geq 3\left(\frac{4}{3}\sqrt{3}F\right)=4\sqrt{3}F$$

Equality holds if and only if $\Delta ABC$ is equilateral.