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S.2410 In ΔABC the following relationship holds:

$$\frac{r_a}{\sqrt{r_b r_c}}\cdot bc+\frac{r_b}{\sqrt{r_c r_a}}\cdot ca+\frac{r_c}{\sqrt{r_a r_b}}\cdot ab\ge 4\sqrt{3}F$$

Proposed by D.M. Bătinețu-Giurgiu, Claudia Nănuți – Romania

Solution by Titu Zvonaru-Romania

Using $AM-GM$ inequality and Carlitz's inequality $(abc)^{2/3}\ge \frac{4}{3}F\sqrt{3}$, we obtain:

$$\frac{r_a}{\sqrt{r_b r_c}}\cdot bc+\frac{r_b}{\sqrt{r_c r_a}}\cdot ca+\frac{r_c}{\sqrt{r_a r_b}}\cdot ab\ge$$

$$\ge 3\left(\frac{r_a}{\sqrt{r_b r_c}}\cdot\frac{r_b}{\sqrt{r_c r_a}}\cdot\frac{r_c}{\sqrt{r_a r_b}}a^2b^2c^2\right)^{1/3}\ge 3\left(\frac{4}{3}\sqrt{3}F\right)=4\sqrt{3}F.$$

Equality if and only if ΔABC is equilateral .