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**S.2412** If $m > 0$ and $M \in Int(\Delta ABC), d_a = d(M, BC), d_b = d(M, CA)$, $d_c = d(M, AB)$ then:

$$\frac{ab^m}{d_a^m h_b} + \frac{bc^m}{d_b^m h_c} + \frac{ca^m}{d_c^m h_a} \geq \left(2\sqrt{3}\right)^{m+1}$$

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We have $ad_a + bd_b + cd_c = 2F$.

By $AM - GM$ inequality we have $\left((ad_a)(bd_b)(cd_c)\right) \leq \frac{(ad_a+bd_b+cd_c)^3}{27} = \frac{8F^3}{27}$.

Using $AM - GM$ inequality and Carlitz's inequality $(abc)^{2/3} \geq \frac{4}{3}\sqrt{3}F$, it follows that:

$$\frac{ab^m}{d_a^m h_b} + \frac{bc^m}{d_b^m h_c} + \frac{ca^m}{d_c^m h_a} = \frac{a^{m+1}b^{m+1}}{a^m d_a^m b h_b} + \frac{b^{m+1}c^{m+1}}{b^m d_b^m c h_c} + \frac{c^{m+1}a^{m+1}}{c^m d_c^m a h_a} \geq$$

$$\geq \frac{3}{F}\left(\frac{a^{2m+2}b^{2m+2}c^{2m+2}}{\left((ad_a)(bd_b)(cd_c)\right)^m}\right)^{\frac{1}{3}} \geq \frac{3}{F} \cdot \frac{4^{m+1}}{3^{m+1}}\left(\sqrt{3}F\right)^{m+1} \cdot \frac{3^m}{2^m F^m} = \left(2\sqrt{3}\right)^{m+1}$$

Equality if and only if $\Delta ABC$ is equilateral and $M$ is the circumcenter.