ROMANIAN MATHEMATICAL MAGAZINE

S.2413 If $m\geq 0, u, v>0$ and $X\in \text{Int}(\Delta ABC), x_a=d(M,BC)$,

$x_b=d(M,CA), x_c=d(M,AB)$, then:

$$\frac{a}{(uh_a+vx_a)^m}+\frac{b}{(uh_b+vx_b)^m}+\frac{c}{(uh_c+vx_c)^m}\geq \frac{6\sqrt{3}}{(3u+v)^m\cdot r^{m-1}}$$

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We have $ah_a=bh_b=ch_c=2F$ and $ax_a+bx_b+cx_c=2F$.

Applying Radon's inequality and Child's inequality $s\geq 3\sqrt{3}r$, we obtain:

$$\frac{a}{(uh_a+vx_a)^m}+\frac{b}{(uh_b+vx_b)^m}+\frac{c}{(uh_c+vx_c)^m}=$$

$$=\frac{a^{m+1}}{(2uF+vax_a)^m}+\frac{b^{m+1}}{(2uF+vbx_b)^m}+\frac{c^{m+1}}{(2uF+vcx_c)^m}\geq$$

$$\geq \frac{(a+b+c)^{m+1}}{\left(6uF+v(ax_a+bx_b+cx_c)\right)^m}=\frac{2^{m+1}s^{m+1}}{2^mF^m(3u+v)^m}=$$

$$=\frac{2s}{(3u+v)^mr^m}\geq \frac{6\sqrt{3}}{(3y+v)^mr^{m-1}}$$

Equality holds if and only if ΔABC is equilateral.