ROMANIAN MATHEMATICAL MAGAZINE

S.2415 If $x, y > 0$ and in ΔABC, $2x\cdot s > y\cdot \max\{a,b,c\}$, then

$$\frac{a^3}{2xs-ya}+\frac{b^3}{2xs-yb}+\frac{c^3}{2xs-yc}\ge\frac{4\sqrt{3}F}{3x-y}$$

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Solution by Titu Zvonaru-Romania

Using Bergström inequality, the known inequality $(a+b+c)^2\le 3(a^2+b^2+c^2)$ and Ionescu – Weitzenböck inequality $a^2+b^2+c^2\ge 4\sqrt{3}F$, we obtain:

$$\frac{a^3}{2xs-ya}+\frac{b^3}{2xs-yb}+\frac{c^3}{2xs-yc}=$$

$$=\frac{a^4}{2xas-ya^2}+\frac{b^4}{2xbs-yb^2}+\frac{c^4}{2xcs-yc^2}\ge$$

$$\ge\frac{(a^2+b^2+c^2)^2}{x(a+b+c)^2-y(a^2+b^2+c^2)}\ge\frac{(a^2+b^2+c^2)^2}{(3x-y)(a^2+b^2+c^2)}=$$

$$=\frac{a^2+b^2+c^2}{3x-y}\ge\frac{4\sqrt{3}F}{3x-y}.$$

Equality holds if and only if ΔABC is equilateral.