ROMANIAN MATHEMATICAL MAGAZINE

S.2417 If $M \in Int(\Delta ABC), d_a = d(M, BC), d_b = d(M, CA), d_c = d(M, AB)$ then:

$$\frac{a^2b}{d_b + h_b} + \frac{b^2c}{d_c + h_c} + \frac{c^2a}{d_a + h_a} \geq 6F$$

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Solution by Titu Zvonaru-Romania

Let F b the area of ΔABC. We have $ah_a = bh_b = ch_c = 2F, ad_a + bd_b + cd_c = 2F$.

Using Bergström inequality and Gordon's inequality, it follows that

$$\frac{a^2b}{d_b + h_b} + \frac{b^2c}{d_c + h_c} + \frac{c^2a}{d_a + h_a} =$$

$$= \frac{a^2b^2}{bd_b + bh_b} + \frac{b^2c^2}{cd_c + ch_c} + \frac{c^2a^2}{ad_a + ah_a} \geq$$

$$\geq \frac{(ab + bc + ca)^2}{8F} \geq \frac{48F^2}{8F} = 6F$$

Equality holds for: $a = b = c$.