ROMANIAN MATHEMATICAL MAGAZINE

S.2420 If $M \in Int(\Delta ABC), x = [MBC], y = [MCA], z = [MAB]$ then:

$$(x^{1+x} + y^{1+x} + z^{1+x}) + (x^{1+y} + y^{1+y} + z^{1+y}) + (x^{1+z} + y^{1+z} + z^{1+z}) \ge 3^{1-\frac{F}{3}} \cdot F^{1+\frac{F}{3}}$$

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We have $x + y + z = F$. By Power Mean inequality we obtain:

$$\left(\frac{x^{1+x} + y^{1+x} + z^{1+x}}{3}\right)^{\frac{1}{1+x}} \ge \frac{x+y+z}{3} \Leftrightarrow 3^x(x^{1+x} + y^{1+x} + z^{1+x}) \ge F^{1+x} \quad (1)$$

Applying (1) and $AM - GM$ inequality it follows that

$$(x^{1+x} + y^{1+x} + z^{1+x}) + (x^{1+y} + y^{1+y} + z^{1+y}) + (x^{1+z} + y^{1+z} + z^{1+z}) \ge$$

$$\ge \frac{F^{1+x}}{3^x} + \frac{F^{1+y}}{3^y} + \frac{F^{1+z}}{3^z} \ge 3\left(\frac{F^{1+x}}{3^x} \cdot \frac{F^{1+y}}{3^y} \cdot \frac{F^{1+z}}{3^z}\right)^{\frac{1}{3}} = 3^{1-\frac{x+y+z}{3}} \cdot F^{1+\frac{x+y+z}{3}} = 3^{1-\frac{F}{3}} \cdot F^{1+\frac{F}{3}}$$

Equality holds if and only if M centroid of ΔABC.