ROMANIAN MATHEMATICAL MAGAZINE

S.2425 If $n, k, a, b, c > 0$ then:

$$\frac{a^3}{nab^2+kc^3}+\frac{b^3}{nbc^2+ka^3}+\frac{c^3}{nca^2+kb^3}\geq\frac{3}{n+k}$$

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Solution by Titu Zvonaru-Romania

Using Bergström inequality, Cârtoaje's inequality, and the known inequality:

$(x+y+z)^2 \geq 3(xy+yz+zx)$, **we obtain:**

$$\frac{a^3}{nab^2+kc^3}+\frac{b^3}{nbc^2+ka^3}+\frac{c^3}{nca^2+kb^3}=$$

$$=\frac{a^4}{na^2b^2+kc^3a}+\frac{b^4}{nb^2c^2+ka^3b}+\frac{c^4}{nc^2a^2+kb^3c}=$$

$$\geq\frac{(a^2+b^2+c^2)^2}{n(a^2b^2+b^2c^2+c^2a^2)+k(a^3b+b^3c+c^3a)}\geq$$

$$\geq\frac{3(a^2+b^2+c^2)^2}{(n+k)(a^2+b^2+c^2)^2}=\frac{3}{n+k}$$