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S.2434 In the triangle ABC

$$\sum_{cyc} h_b h_c + \sum_{cyc} (s-b)(s-c) \le \frac{4}{3} \sum_{cyc} m_a^2$$

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Using the inequality $4(h_a h_b + h_b h_c + h_c h_c) \le 3(ab + bc + ca)$ (item 6.4 from $[1]$)

and $m_a^2 + m_b^2 + m_c^2 = \frac{3(a^2+b^2+c^2)}{4}$,

the inequality is equivalent to

$$\sum_{cyc} ab + \sum_{cyc} (a+b-c)(a-b+c) \le \sum_{cycc} a^2 \Leftrightarrow ab + bc + ca \le a^2 + b^2 + c^2.$$

Equality holds if and only if the triangle ABC is equilateral.

$[1]$ O. Bottema, *Geometric Inequalities*, Groningen 1969