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S.2462 If $x, y, z \geq 0$, then in ΔABC holds:

$$\frac{ayz}{h_a}+\frac{byz}{h_b}+\frac{czx}{h_c}\leq\frac{R^2}{2F}(x+y+z)^2$$

Proposed by D.M.Bătinețu-Giurgiu, Claudia Nănuți – Romania

Solution by Titu Zvonaru-Romania

Since $ah_a = bh_b = ch_c = 2F$, we have

$$\frac{ayz}{h_a}+\frac{byz}{h_b}+\frac{czx}{h_c}=\frac{a^2yz+b^2zx+c^2xy}{2F}.$$

We have to prove that $a^2yz+b^2zx+c^2xy\leq R^2(x+y+z)^2$.

This inequality is the inequality of Kooi.

Equality holds if and only if ΔABC is equilateral and $x=y=z$.

Note by editor:

The solver used Klamkin's inequality (1975):

If R_1, R_2, R_3 –are the distances from a point P to the vertices A, B, C of ΔABC with sides a, b, c then:

$$a^2yz+b^2zx+c^2xy\leq(x+y+z)\left(xR_1^2+yR_2^2+zR_3^2\right)$$

If we take in (1), $R_1=R_2=R_3=R$ which means that $P=O$ –the circumcenter of ΔABC

we obtain Kooi's inequality:

$$a^2yz+b^2zx+c^2xy\leq R^2(x+y+z)^2$$