ROMANIAN MATHEMATICAL MAGAZINE

S.2463 If $m \geq 0$ and $x, y, z > 0$, then in ΔABC holds:

$$\frac{x^{m+1}a^2}{(y+z)^{m+1}} + \frac{y^{m+1}b^2}{(z+x)^{m+1}} + \frac{z^{m+1}c^2}{(x+y)^{m+1}} \geq 2^{1-m}\sqrt{3} \cdot F$$

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By Power Mean inequality we have

$$\left(\frac{y^{m+1} + z^{m+1}}{2}\right)^{\frac{1}{m+1}} \geq \frac{y+z}{2} \Leftrightarrow (y+z)^{m+1} \leq 2^m\left(y^{m+1} + z^{m+1}\right) \qquad (1)$$

Applying (1) and Tsintsifas' inequality

$\frac{x}{y+z}a^2 + \frac{y}{z+x}b^2 + \frac{z}{x+y}c^2 \geq 2\sqrt{3} \cdot F$, it follows that

$$\frac{x^{m+1}a^2}{(y+z)^{m+1}} + \frac{y^{m+1}b^2}{(z+x)^{m+1}} + \frac{z^{m+1}c^2}{(x+y)^{m+1}} \geq$$

$$\geq \frac{1}{2^m}\left(\frac{x^{m+1}}{y^{m+1} + z^{m+1}}a^2 + \frac{y^{m+1}}{z^{m+1} + x^{m+1}}b^2 + \frac{z^{m+1}}{x^{m+1} + y^{m+1}}c^2\right) \geq$$

$$\geq \frac{1}{2^m}\left(2\sqrt{3} \cdot F\right) = 2^{1-m}\sqrt{3} \cdot F.$$

Equality holds if and only if ΔABC is equilateral and $x = y = z$.