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S.2464 If $m > 0$ and $M \in Int(\Delta ABC), d_a = d(M, BC), d_b = d(M, CA),$ $d_c = d(M, AB)$ then:

$$\frac{a^{2m+1}}{d_a} + \frac{b^{2m+1}}{d_b} + \frac{c^{2m+1}}{d_c} \geq 2^{2m+1}\left(\sqrt{3}\right)^{3-m} F^m.$$

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Solution by Titu Zvonaru-Romania

We have $ad_a + bd_b + cd_c = 2F$, then by AM-GM: $\left((ad_a)(bd_b)(cd_c)\right) \leq \frac{8F^3}{27}$.

Using $AM - GM$ inequality and Carlitz's inequality $(abc)^{2/3} \geq \frac{4}{3}\sqrt{3}F$ it follows that

$$\frac{a^{2m+1}}{d_a} + \frac{b^{2m+1}}{d_b} + \frac{c^{2m+1}}{d_c} = \frac{a^{2m+2}}{ad_a} + \frac{b^{2m+2}}{bd_b} + \frac{c^{2m+2}}{cd_c} \geq$$

$$\geq 3\left(\frac{4}{3}\sqrt{3}F\right)^{m+1} \frac{3}{2F} = 2^{2m+1}\left(\sqrt{3}\right)^{3-m} F^m$$