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S.2465 If $m > 0$ then in ΔABC holds:

$$\frac{a^{m+2}}{h_a^m}+\frac{b^{m+2}}{h_b^m}+\frac{c^{m+2}}{h_c^m}\geq 2^{m+2}\left(\sqrt{3}\right)^{1-m}F$$

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We have $ah_a=bh_b=ch_c=2F$. Using $AM-GM$ inequality and Carlitz's inequality

$(abc)^{2/3}\geq\frac{4}{3}\sqrt{3}F$, it follows that:

$$\frac{a^{m+2}}{h_a^m}+\frac{b^{m+2}}{h_b^m}+\frac{c^{m+2}}{h_c^m}=\frac{a^{2m+2}}{a^m h_a^m}+\frac{b^{2m+2}}{b^m h_b^m}+\frac{c^{2m+2}}{c^m h_c^m}\geq$$

$$\geq 3\left(\frac{4}{3}\sqrt{3}F\right)^{m+1}\frac{1}{2^m F^m}=2^{m+2}\left(\sqrt{3}\right)^{1-m}F^m.$$

Equality holds for $a=b=c$.