S.2467 If $x,y,z>0$, then in ΔABC holds:

$$\frac{a^4e^{x^2}}{y+z}+\frac{b^4e^{y^2}}{z+x}+\frac{c^4e^{z^2}}{x+y}>16F^2$$

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For $t>0$ we consider the function $f(t)=e^t-1-t$.

We have $f'(t)=e^t-1, f''(t)=e^t>0$. Yields that f' is an increasing function, hence $f'(t)>f'(0)=0$. We deduce that the function f is increasing;

then $f(t)>f(0)=0$. For $t=x^2$, by $AM-GM$ it results that $e^{x^2}>1+x^2\geq 2x \quad (1)$

Applying inequality (1), Bergström's inequality and the Heron's formula

$16F^2=2(a^2b^2+b^2c^2+c^2a^2)-(a^4+b^4+c^4)$, it follows that

$$\frac{a^4e^{x^2}}{y+z}+\frac{b^4e^{y^2}}{z+x}+\frac{c^4e^{z^2}}{x+y}>2\left(\frac{xa^4}{y+z}+\frac{yb^4}{z+x}+\frac{zc^4}{x+y}\right)=$$

$$=2\left(\frac{xa^4}{y+z}+a^4+\frac{yb^4}{z+x}+b^4+\frac{zc^4}{x+y}+c^4-(a^4+b^4+c^4)\right)=$$

$$=2\left(\frac{(x+y+z)a^4}{y+z}+\frac{(x+y+z)b^4}{z+x}+\frac{(x+y+z)c^4}{x+y}-(a^4+b^4+c^4)\right)\geq$$

$$\geq 2\left(\frac{(x+y+z)(a^2+b^2+c^2)^2}{y+z+z++x+y}-(a^4+b^4+c^4)\right)=(a^2+b^2+c^2)^2-2(a^4+b^4+c^4)=$$

$$=2(a^2b^2+b^2c^2+c^2a^2)-(a^4+b^4+c^4)=16F^2.$$