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S.2481 AB **diameter in a semicircle with** O **– center.** $ACDO$ **– is a square outside semicircle,** E **– is a point on semicircle such that** $m(< CBE) = 90°$**.**
Prove that : $[ACDO] = [CBE]$**.**

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Solution by Titu Zvonaru-Romania

Suppose that $AO = 1$. Let M the orthogonal projection of A onto BC. Since E lies on the semicircle, we have:

$$< AEB =< EBC =< BMA.$$

It results that the quadrilateral $AMBE$ has three right angles, hence $AMBE$ is a rectangle.

From the right-angled triangle ABC we obtain

$$AC = 1, AB = 2, BC = \sqrt{1+4} = \sqrt{5}, AM = \frac{AC \cdot AB}{BC} = \frac{2}{\sqrt{5}}.$$

Since $EB = AM$ and $m(< CBE) = 90°$, it follows that

$$[CBE] = \frac{BC \cdot EB}{2} = \frac{\sqrt{5}}{2} \cdot \frac{2}{\sqrt{5}} = 1 = [ACDO]$$