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S.2522 If $x, y > 0$ and $M \in Int(\Delta ABC)$ $F_a = [MBC], F_b = [MCA]$, $F_c = [MAB]$, then:

$$\frac{a^4}{xF_b + yF_c} + \frac{b^4}{xF_c + yF_a} + \frac{c^4}{xF_a + yF_b} \geq \frac{48}{x+y} \cdot F$$

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Solution by Titu Zvonaru-Romania

We have $F_a + F_b + F_c = F$. Applying Bergström's inequality and Ionescu-Weitzenbock's inequality $a^2 + b^2 + c^2 \geq 4\sqrt{3}F$, it follows that:

$$\frac{a^4}{xF_b + yF_c} + \frac{b^4}{xF_c + yF_a} + \frac{c^4}{xF_a + yF_b} \geq$$

$$\geq \frac{(a^2+b^2+c^2)^2}{x(F_a+F_b+F_c) + y(F_a+F_b+F_c)} \geq \frac{\left(4\sqrt{3}F\right)^2}{(x+y)F} = \frac{48}{x+y} \cdot F^2$$

Equality holds if and only if the triangle ABC is equilateral.