ROMANIAN MATHEMATICAL MAGAZINE

S.2530 Solve for real numbers:

$$x + y = 18$$

$$u + v = 15$$

$$xy = uv$$

$$x^2 + y^2 + u^2 + v^2 = 325$$

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Solution by Titu Zvonaru-Romania

The fourth equation is equivalent to

$$(x+y)^2 - 2xy + (u+v)^2 - 2uv = 325 \Leftrightarrow$$

$$18^2 - 4xy + 15^2 = 325 \Leftrightarrow 4xy = 324 + 225 - 325 \Leftrightarrow xy = 56.$$

By $x + y = 18, xy = 56$ we obtain the quadratic equation $t^2 - 18t + 56 = 0$, hence $\{x, y\} = \{4, 14\}$. By $u + v = 15, uv = 56$ and the quadratic equation $t^2 - 15t + 56 = 0$ yields $\{u, v\} = \{7, 8\}$.