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S.2557 If $m, n, p, x, y, z > 0$ then in any triangle ABC holds:

$$\frac{mx+ny}{pz}\cdot a^2+\frac{ny+pz}{mx}\cdot b^2+\frac{pz+mx}{ny}\cdot c^2\geq 8\sqrt{3}\cdot F$$

Proposed by D.M.Bătinețu-Giurgiu – Romania

Solution by Titu Zvonaru-Romania

Applying $AM-GM$ and Gordon's inequality $ab+bc+ca\geq 4\sqrt{3}\cdot F$, it follows that

$$\frac{mx+ny}{pz}\cdot a^2+\frac{ny+pz}{mx}\cdot b^2+\frac{pz+mx}{ny}\cdot c^2=$$

$$=\left(\frac{mx}{pz}\cdot a^2+\frac{pz}{mx}\cdot b^2\right)+\left(\frac{ny}{pz}\cdot a^2+\frac{pz}{ny}\cdot c^2\right)+\left(\frac{ny}{mx}\cdot b^2+\frac{mx}{ny}\cdot c^2\right)\geq$$

$$\geq 2(ab+bc+ca) => 8\sqrt{3}\cdot F.$$

Equality holds if and only if the triangle ABC is equilateral and $mx=ny=pz$.