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S.2565 Solve for real numbers:

$$\sqrt{3}(2x+2y+1)=2\sqrt{x^2+y^2+1}+\sqrt{2x^2+1}+2\sqrt{x^2+2y^2}.$$

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Applying the known inequality $3\left(a^2+b^2+c^2\right)\geq(a+b+c)^2$, it follows that

$$3(2x+2y+1)=2\sqrt{3(x^2+y^2+1)}+\sqrt{3(x^2+x^2+1)}+2\sqrt{3(x^2+y^2+y^2)}\geq$$

$$\geq 2(x+y+1)+(x+x+1)+2(x+y+y)=3(2x+2y+1)$$

We deduce that there is the equality, hence $x=y=1$.

It easy to check that $x=y=1$ verify the given equation.