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S.2571 If $m \geq 0, x, y > 0$ and let T, U be two interior points in triangle ABC, $t_a = d(T, BC), t_b = d(T, CA), t_c = d(T, AB)$ and similar u_a, u_b, u_c, then

$$\frac{a^{m+1}b}{(xt_b + yu_b)^m} + \frac{b^{m+1}c}{(xt_c + yu_c)^m} + \frac{c^{m+1}a}{(xt_a + yu_a)^m} \geq \frac{2^{m+2}\left(\sqrt{3}\right)^{m+1}}{(x+y)^m} \cdot F$$

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Solution by Titu Zvonaru-Romania

We have $at_a + bt_b + ct_c = au_a + bu_b + cu_c = 2F$.

Applying Radon's inequality and Gordon's inequality $ab + bc + ca \geq 4\sqrt{3}F$,

it follows that

$$\frac{a^{m+1}b}{(xt_b + yu_b)^m} + \frac{b^{m+1}c}{(xt_c + yu_c)^m} + \frac{c^{m+1}a}{(xt_a + yu_a)^m} =$$

$$= \frac{a^{m+1}b^{m+1}}{(xbt_b + ybu_b)^m} + \frac{b^{m+1}c^{m+1}}{(xct_c + ycu_c)^m} + \frac{c^{m+1}a^{m+1}}{(xat_a + yau_a)^m} \geq$$

$$\geq \frac{(ab + bc + ca)^{m+1}}{\left(x(bt_b + ct_c + at_a) + y(bu_b + cu_c + au_a)\right)^m} \geq \frac{\left(4\sqrt{3}F\right)^{m+1}}{(2F)^m(x+y)^m} = \frac{2^{m+2}\left(\sqrt{3}\right)^{m+1}}{(x+y)^m} \cdot F.$$

Equality holds if and only if the triangle ABC is equilateral.