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S.2577 If $u, v, x, y, z > 0$ and $f: \mathbb{R}_+^* \rightarrow \mathbb{R}_+^*$, then

$$\frac{x}{uy + vf(x, y)} + \frac{y}{uf(x, y) + vx} + \frac{f(x, y)}{ux + vy} \geq \frac{3}{u + v}$$

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Apply Bergström's inequality and the known inequality $(a + b + c)^2 \geq 3(ab + bc + ca)$,

it follows that

$$\begin{aligned} & \frac{x}{uy + vf(x, y)} + \frac{y}{uf(x, y) + vx} + \frac{f(x, y)}{ux + vy} = \\ &= \frac{x^2}{uxy + vxf(x, y)} + \frac{y^2}{uyf(x, y) + vxy} + \frac{(f(x, y))^2}{uxf(x, y) + vyf(x, y)} \geq \\ & \geq \frac{(x + y + f(x, y))^2}{(u + v)(xy + xf(x, y) + yf(x, y))} \geq \frac{3}{u + v}. \end{aligned}$$

Equality holds if and only if $x = y = f(x, y)$.