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S.2578 If $m \geq 0, t, x, y, z > 0$ then in triangle ABC holds:

$$\frac{y^{t+1} + z^{t+1}}{x^{t+1}} a^{m+1} + \frac{z^{t+1} + x^{t+1}}{y^{t+1}} b^{m+1} + \frac{x^{t+1} + y^{t+1}}{z^{t+1}} c^{m+1} \geq 2^{m+2} 3^{\frac{3-m}{4}} (\sqrt{F})^{m+1}$$

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We denote $m = x^{t+1}, n = y^{t+1}, p = z^{t+1}$. Applying $AM - GM$ inequality, Cesaro's inequality $(m+n)(n+p)(p+m) \geq 8mnp$ and Carlitz's inequality $(abc)^{\frac{2}{3}} \geq \frac{4}{3}\sqrt{3F}$,

it follows that

$$\begin{aligned} & \frac{y^{t+1} + z^{t+1}}{x^{t+1}} a^{m+1} + \frac{z^{t+1} + x^{t+1}}{y^{t+1}} b^{m+1} + \frac{x^{t+1} + y^{t+1}}{z^{t+1}} c^{m+1} = \\ & = \frac{n+p}{m} a^{m+1} + \frac{p+m}{n} b^{m+1} + \frac{m+n}{p} c^{m+1} \geq \\ & \geq 3 \left(\frac{n+p}{m} a^{m+1} \cdot \frac{p+m}{n} b^{m+1} \cdot \frac{m+n}{p} c^{m+1} \right)^{\frac{1}{3}} \geq \\ & \geq 3(8(abc)^{m+1})^{\frac{1}{3}} \geq 3 \cdot 2 \left(\frac{4}{3}\sqrt{3F} \right)^{\frac{m+1}{2}} = 2^{m+2} 3^{\frac{3-m}{4}} (\sqrt{F})^{m+1}. \end{aligned}$$

Equality holds if and only if the triangle ABC is equilateral and $x = y = z$.