

ROMANIAN MATHEMATICAL MAGAZINE

S.2602 If $m \geq 0$ and $x, y, z > 0$ then in the triangle ABC the following relationship holds:

$$\frac{mx + (m + 1)y}{(m + 1)x + my + (2m + 1)z} a^2 + \frac{my + (m + 1)z}{(m + 1)y + mz + (2m + 1)x} b^2 + \frac{mz + (m + 1)x}{(m + 1)z + mx + (2m + 1)y} c^2 \geq 2\sqrt{3} \cdot F$$

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We denote $u = my + (m + 1)z, v = mz + (m + 1)x, w = mx + (m + 1)y$. We get:

$$u + v = (m + 1)x + my + (2m + 1)z, v + w = (2m + 1)x + (m + 1)y + mz,$$

$$w + u = mx + (2m + 1)y + (m + 1)z.$$

The given inequality is equivalent to

$$\frac{w}{u + v} a^2 + \frac{u}{v + w} b^2 + \frac{v}{w + u} c^2 \geq 2\sqrt{3} \cdot F \quad (1)$$

The inequality (1) is Tsintsifas inequality.

Equality holds if and only the triangle ABC is equilateral and

$$u = v = w \text{ (that is } x = y = z.)$$