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S.2603 If $t \geq 0, x, y > 0$ and M an inner point of triangle ABC,

$d_a = d(M, BC), d_b = d(M, CA), d_c = d(M, AB)$ then:

$$\frac{a^{t+1} \cdot b}{(xd_b + yh_b)} + \frac{b^{t+1} \cdot c}{(xd_c + yh_c)} + \frac{c^{t+1} \cdot a}{(xd_a + yh_a)} \geq \frac{2^{t+2}\left(\sqrt{3}\right)^{t+1}}{(x+3y)^t} \cdot F$$

Proposed by D.M.Bătinețu-Giurgiu, Carmen Terheci – Romania

Solution by Titu Zvonaru-Romania

We have $ah_a = bh_b = ch_c = 2F, ad_a + bd_b + ch_c = 2F$.

Applying Radon's inequality and Gordon's inequality $ab + bc + ca \geq 4\sqrt{3}F$,

it follows that

$$\frac{a^{t+1} \cdot b}{(xd_b + yh_b)} + \frac{b^{t+1} \cdot c}{(xd_c + yh_c)} + \frac{c^{t+1} \cdot a}{(xd_a + yh_a)} =$$

$$= \frac{a^{t+1} \cdot b^{t+1}}{(xbd_b + ybh_b)} + \frac{b^{t+1} \cdot c^{t+1}}{(xcd_c + ych_c)} + \frac{c^{t+1} \cdot a^{t+1}}{(xad_a + yah_a)} \geq$$

$$\geq \frac{(ab + bc + ca)^{t+1}}{(x(ad_a + bd_b + cd_c) + y(ah_a + bh_b + ch_c))^t} = \frac{(ab + bc + ca)^{t+1}}{(2xF + 6yF)^t} \geq$$

$$\geq \frac{\left(4\sqrt{3}F\right)^{t+1}}{(2F)^t (x+3y)^t} = \frac{2^{t+2}\left(\sqrt{3}\right)^{t+1}}{(x+3y)^t} \cdot F$$

Equality holds if and only if the triangle ABC is equilateral.