S.2639 If $m, n > 0$ and $m + n = mn$ then in any triangle ABC with the area F the following inequality holds:

$$(a^m + b^m + c^m)^{\frac{1}{m}}(a^n + b^n + c^n)^{\frac{1}{n}} \geq 4\sqrt{3}F$$

Proposed by D.M.Bătinețu-Giurgiu, Daniel Sitaru – Romania

Solution by Titu Zvonaru-Romania

By Power Mean inequality we have:

$$\left(\frac{a^m + b^m + c^m}{3}\right)^{1/m} \geq \frac{a+b+c}{3} \Leftrightarrow (a^m + b^m + c^m)^{\frac{1}{m}} \geq \frac{3^{\frac{1}{m}}(a+b+c)}{3} \quad (1)$$

$$\left(\frac{a^n + b^n + c^n}{3}\right)^{1/n} \geq \frac{a+b+c}{3} \Leftrightarrow (a^n + b^n + c^n)^{\frac{1}{n}} \geq \frac{3^{\frac{1}{n}}(a+b+c)}{3} \quad (2)$$

Since $m + n = mn \Leftrightarrow \frac{1}{m} + \frac{1}{n} = 1$, applying $(1), (2)$ and the inequality:

$s^2 \geq 3F\sqrt{3}$ (Mitrinovic), it follows that:

$$(a^m + b^m + c^m)^{\frac{1}{m}}(a^n + b^n + c^n)^{\frac{1}{n}} \geq \frac{3^{\frac{1}{m}+\frac{1}{n}}(a+b+c)^2}{9} \geq \frac{12F\sqrt{3}}{3} = 4\sqrt{3}F.$$

Equality holds if and only if the triangle ABC is equilateral.